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ABSTRACT

The results of a study of student attitudes toward mathematics before and after studying a "Mathematics Through Science" unit in the eighth grade are given in this report. Six significant attitude differences were found and used to group students into eight attitude groups. The identifying characteristics of these groups are discussed in detail. The number of teachers represented in each group is considered, as are the number of boys and girls in each group. Mean scores of the groups on ability, algebra achievement, and attitude are compared. No significant differences for any of these scales were found which hold for both of the samples considered. It was concluded that the formation of strong cohesive attitudes-groups is not a major factor for consideration in the design of mathematics units taught via physical materials. This document previously announced as ED 042 631. (Author/FL)

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SMSG REPORTS

No. 8

The Mathematics Through Science Study: Attitude Changes in a Mathematics Laboratory

Jon L. Higgins

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THE MATHEMATICS THROUGH SCIENCE STUDY:

ATTITUDE CHANGES IN A MATHEMATICS LABORATORY

Summary

This report gives the results of a study of student attitudes toward mathematics before and after studying a Mathematics Through Science unit in the eighth grade. Six significant attitude differences are found and used to group students into naturally occurring attitude groups. The identifying characteristics of these groups are discussed in detail. Of the eight groups formed, the largest are characterized by lack of attitude changes. Favorable and unfavorable groups, as well as groups which appear to favor only some aspects of the unit, account for only a minority of students. These groups range in size from 3 percent to 10 percent of the samples considered.

The number of teachers represented in each group is considered, as are the number of boys and girls in each group. No teacher is inordinately represented in any group, but one group (which finds the unit more interesting and more difficult) is found to be primarily composed of boys.

Mean scores of groups on ability, achievement (algebra) and attitude are compared. No significant differences for any of these scales are found which hold for both of the samples considered. We conclude that the formation of strong cohesive attitude-groups is not a major factor for consideration in the design of mathematics units taught via physical materials.

Introduction

Most mathematics educators agree on the importance of using physical materials to approach mathematical abstractions in the early elementary grades. Whether or not such an approach should be continued in the middle and upper grades is currently an open question. In the summer of 1963 School Mathematics Study Group brought together a group of mathematicians, scientists, and teachers to prepare sample materials for grades seven, eight, and nine which would utilize such an approach. The results of this writing team's efforts were three Mathematics Through Science (MTS) units which could be used as supplements to the usual course work at the junior high level. During the 1963-1964 school year the MTS units were tried out in 90 schools representing Massachusetts, Connecticut, Delaware, the District of Columbia, Virginia, Oklahoma, Texas and California. 5,192 seventh grade students,

5,310 eighth grade students, 2,009 ninth grade students and a total of 210 teachers were involved.

A revision team revised the units in the summer of 1964, basing their changes on the written evaluations of the field teachers. Much of the revision centered on efforts to make the vocabulary of the text materials more suitable for junior high school students. Experiments were revised or replaced where evaluations indicated student or equipment difficulties. Some modifications were made in the sequence of experiments in order to enhance concept continuity. The revised units and their suggested grade levels are: Measurement and Graphing (grade seven); Graphing, Equations and Linear Functions (grade eight); and An Experimental Approach to Function (grade nine).

Teacher reports of student acceptance of the units during the field trials were extremely mixed. In particular, many teachers felt that this approach worked better with some students than with others. Eighth-grade students were less impressed with Part II than were seventh-grade students with Part I. Some teachers reported an upward surge in student interest while studying the units; others reported that students were confused by the scientific principles involved.

Design of the Study

In the spring of 1968 School Mathematics Study Group undertook a more formal study and evaluation of student responses to one of the Mathematics Through Science units. The eighth grade unit (Graphing, Equations and Linear Functions) was chosen because earlier evaluations had raised more uncertainty about student response to this unit than to either of the others. Teachers' comments had suggested that distinct interest-groupings of students occurred during this time; hence it was surmised that proper testing and analysis might show the existence of these groups statistically. Instead of formal hypothesis-testing the study was designed to explore the nature of such groups and their relationship to student achievement.

Twenty-nine eighth-grade mathematics teachers from junior high schools in Santa Clara County, California, agreed to teach the unit to one of their classes. The schools were all located in suburban areas characterized by a majority of middle socio-economic class families. Descriptive measures of teacher and class populations are included in Appendix I. Classes contained an average of 30 students and met daily for an average length of 46 minutes. The standard text for most classes was Mathematics 8 by McSwain, Brown, Gundlach, and Cooke published by Laidlaw Brothers. However, eight

classes used Exploring Modern Mathematics, Book 2 by Keedy, Jameson and Johnson published by Holt, Rinehart and Winston. One class used an earlier version (1955) of the Laidlaw text, entitled Understanding Arithmetic 8, by McSwain, Cooke, and Ulrich.

In a very general sense the text used indicates an estimation of the class ability, with Exploring Modern Mathematics being used with higher (estimated) ability students, and Understanding Arithmetic used by lower (estimated) ability students. All classes had covered text material dealing with the rational number system (including positive and negative numbers) but had not worked with any of the basic concepts contained in the Mathematics Through Science unit.

Most teachers had 5-9 years teaching experience, although four had taught more than 13 years. Twenty-two of the twenty-nine teachers had previously had experience teaching a science class, although most of those indicated that this had been a course at the elementary school level. Only eight were currently teaching other subjects in addition to mathematics. Thirteen teachers had had more than ten credits in college mathematics beginning with calculus; twelve teachers had no mathematics credits for calculus or above.

Both text material and laboratory equipment were furnished to the schools by the School Mathematics Study Group. Because we felt the laboratory nature of the unit should be emphasized, materials were supplied on a one per student bases (up to a maximum of 30 per class). The exception to this was the final gas pressure experiment which was performed as a teacher demonstration as the text suggests. Some minor modifications were made in the equipment or procedure specified by the Mathematics Through Science text where these changes would appreciably reduce equipment costs or increase the probability of student success. For example, in Section 1.2 (The Seesaw Experiment) jumbo paper clips were used as weights instead of the suggested standard weight sets. Although the balancing procedure was more sensitive because of the lighter paper clips, the results were more accurate since it is impossible to account for the various effects of the original weight hangers specified in the text. Of course, the substitution also effected a substantial savings in cost. Appendix II contains a complete list of the modifications which were made in equipment and procedures.

Four inservice meetings were held for the teachers in order to acquaint them with the materials and to give them the opportunity to investigate the use of the laboratory equipment themselves before presenting it to the students. Measurements and data were collected for each of the experiments

and discussed in terms of anticipated student responses and difficulties. In addition, the mathematics illustrated by each experiment was discussed. Teachers were paid a small stipend for attendance at these meetings.

A five-week period of time immediately preceeding each school's spring vacation was used for the experimental work. Approximately one week of this period was used for testing, leaving four consecutive weeks for instruction. A test battery was administered immediately before and after the four week instructional period. The battery consisted of three achievement and eighteen attitudinal scales selected from the tests developed for the National Longitudinal Study of Mathematical Abilities. In addition the Necessary Arithmetic Operations scale (NLSMA P4222; French Kit Form R-4, Part I) was administered as an ability measure during the pretreatment testing. All the tests used are described with statistical properties in NLSMA Reports No. 5 according to the identification code shown in table one on the next page. Reliability measures (Cronbach's alpha) for these tests range from 0.59 to 0.85 with the exception of the algebra translation scale whose alpha is 0.38. Actual test items are found in Forms 9151 and 9252 (pp. 209-227), Form 0171 (pp. 311-315) and Forms 5342 and 5243 (pp. 177-203) of NLSMA Reports No. 2.

Initial Comparisons

Table one shows means and standard deviations for the twenty-one scales common to both the pre-treatment and post-treatment test batteries. Each pair of pre- and post-treatment means was compared using a t-ratio for correlated samples (the ratio of the mean of the differences to the standard error of the mean of the differences).

Table 1: Pre- Post-Trial Batteries and Comparisons n = 853

NLSMA Identification	Scale Title	Pre-treatment mean	Pre-treatment σ	Post-treatment mean	Post-treatment σ	t ratio (correlated means)	significance (two-tailed tests)
Y311	Algebra-Number Properties	3.40	1.43	3.76	1.44	8.03	***
Y312	Algebra-Sentences	1.79	1.37	2.22	1.53	9.07	***
Y313	Algebra-Translation	1.01	1.01	1.32	1.08	7.61	***
PY407	Math vs. Non-Math	20.24	4.49	19.93	4.67	-2.66	**
PY408	Math Fun vs. Dull	13.45	4.46	12.82	4.65	-6.03	***
PY409	Pro-Math Composite	33.01	5.85	32.63	6.09	-2.49	*
PY410	Math Easy vs. Hard	26.63	6.83	26.25	6.51	-2.49	*
PY411	Ideal Math Self-Concept	31.56	6.81	29.96	6.98	-9.25	***
PY412	I Think Father Uses Math on Job	4.46	0.96	4.52	0.88	3.10	**
PY413	I Use Math Outside School	3.45	1.14	3.45	1.12	-0.06	n.s.
PY414	I Would Like to Use Math Outside School	3.11	1.02	3.04	1.01	-2.00	*
PY415	Facilitating Anxiety	24.70	5.11	24.67	5.07	-0.28	n.s.
PY416	Debilitating Anxiety	27.70	6.91	27.85	6.98	0.93	n.s.
PY417	Actual Math Self-Concept	30.72	6.98	30.21	6.99	-3.35	***
PY418	Orderliness	36.05	5.96	36.02	6.27	-0.41	n.s.
PY419	Messiness	21.45	5.84	21.79	6.14	2.10	*
PY420	Take More Art	1.94	0.82	1.95	0.83	0.62	n.s.
PY421	Take More Literature	1.67	0.79	1.72	0.79	1.50	n.s.
PY422	Take More Social Studies	2.03	0.81	2.05	0.83	0.75	n.s.
PY423	Take More Math	2.24	0.79	2.17	0.83	-3.19	**
PY424	Take More Science	2.05	0.82	2.03	0.84	-0.36	n.s.

Levels of significance for two-tailed tests:

* $p < .05$, $t = 1.96$ ** $p < .01$, $t = 2.58$ *** $p < .001$, $t = 3.29$

As the table shows, significant gains from pre-treatment to post-treatment were found for all three achievement scales for a two-tailed test judged at the $p < .001$ level.

Using two-tailed tests, significant differences at the $p < .01$ level were found for the attitudinal scales "Mathematics vs. Non-Mathematics," "I Think Father Uses Mathematics on the Job," and "I Would Like to Take More Mathematics." In addition, significant differences at the $p < .001$ level more found for the attitudinal scales "Mathematics Fun vs. Dull," "Ideal Mathematics Self-Concept," and "Actual Mathematics Self-Concept." All six of these attitude scales were scored on what the test developers judged to be a positive attitude toward mathematics. That is, the higher the test score the more favorable the student's attitude toward mathematics. For five of the six significant attitude comparisons post-treatment mean scores were lower than pre-treatment mean scores, an indication of less favorable attitudes toward mathematics after treatment than before treatment.

The sole exception to this trend was the scale "I Think Father Uses Mathematics on the Job." For this scale the post-treatment mean was higher than the pre-treatment mean, an indication that after treatment there was a greater tendancy to feel that father used mathematics on his job. Perhaps this is not surprising since much of the MTS unit relates mathematics to physical devices. On the other hand, the physical devices which are used are balanced meter sticks, bent meter sticks, ball bearings dropping through syrup, bouncing ping-pong balls, and a gas pressure apparatus whose major component was the ball float from a toilet tank valve. It is difficult to imagine students making literal translations from this equipment to the use of mathematics on a job! Apparently many students see mathematical applications connected to the "real world" without undue concern for their practicality.

Grouping Analysis

Because previous evaluation during the pilot testing had indicated that some groups of students responded differently toward the materials than other groups, it was decided to analyze the attitudinal test data for the presence of naturally-occurring groups using a cluster analytic technique. The statistical method chosen for this analysis was Hierarchical Grouping Analysis. Hierarchical grouping is done in a manner which establishes a taxonomy of mutually exclusive sets where each larger unit is a unique combination of previous subordinate units. Development of the procedure was done by

Ward[1], extended to the grouping of profile vectors by Ward and Hook[2], and later generalized for variously defined metrics between groups by Johnson[3].

The computer program used for our grouping was developed by Veldman[4]. This program considers a profile vector of test scores for each individual, and begins by considering each individual as a "group." These groups are then reduced in number by a series of step decisions. At each step some pair of groups is combined, the selection being made so that the total within-groups variance is minimally increased. This increase is printed out as an error term at each step, so that in practice the researcher determines the final number of groups by deciding on the maximum increase in the error term that he will accept between successive steps.

At each step of the grouping program the matrix of potential error terms for each pair of objects must be modified to account for the new group which has just been produced. Because of this the execution time for the program increases prohibitively when a large number of individuals are considered. Thus it was necessary to limit the grouping analysis to a sample of the student population. Since Hierarchical Grouping Analysis is not necessarily predictive, the procedure was carried out independently for two samples of the population. Each sample was formed by randomly selecting four students from each teacher's class, with the exception of three classes which had fewer than twenty students with no missing data on the profile vector. Only three students were randomly selected from each of these classes. Components of the individual profile vectors used as the basis for grouping were the six changes in pre- and post-treatment scores for each of the six attitude scales which had shown significant differences at the .01 or .001 level between pre- and post-treatment administrations. Each component was given equal importance in the program by prestandardizing the raw data matrix by variables

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[1]Ward, J.H. "Hierarchical Grouping to Optimize an Objective Function". Journal of the American Statistical Association, 58, pp. 236-244, 1963.

[2]Ward, J.H. and Hook, M.E. "Application of an Hierarchical Grouping Procedure to a Problem of Grouping Profiles". Educational and Psychological Measurements. 23-1, 1963.

[3]Johnson, Stephen C. "Hierarchical Clustering Schemes". Psychometrika, 32-3, pp. 241-254, Sept. 1967.

[4]Veldman, Donald J. Fortran Programming for the Behavioral Sciences. New York: Holt, Rinehart and Winston, pp. 308-317, 1967.

before computing the initial error matrix. Table 1 of Appendix III compares mean scores on the three algebra achievement scales and mean change scores on the six profile component scales for samples one and two with corresponding means for the total population.

Table 2 of Appendix III shows the error terms for the last nineteen groupings of the program. The grouping procedure for both samples shows fairly regular increases until the step which reduces eight groups into seven. The fact that large error increases occurred with the formation of seven groups for both samples is merely a coincidence. However it does seem to indicate the gross similarity of the two samples. We terminated the grouping procedure with the formation of eight student groups within each sample.

At this point the performance of the eight groups on each of the six pre-treatment attitude scales involved in the profile vector was analyzed using standard univariate analysis of variance techniques. This analysis is especially critical since pre-treatment measures should not be significantly different if a comparison of changes in scores is to be easily interpreted. These statistical tests are included as a part of Appendix IV; the critical scales are the analyses labeled tests 27, 28, 31, 37, 32 and 44. For 7 and 105 degrees of freedom, the critical F-ratio value for $p < .01$ (two-tailed test) is 3.14. Actual F-ratio values do not approach this level except for the pre-treatment "I Think Father Uses Mathematics on the Job" scale for both samples. For this scale the value is 3.89 for sample one and 6.20 for sample two. The mean pre-treatment score for each group of both samples on the "I Think Father Uses Mathematics on the Job" scale is shown in Table 2.

Table 2: Mean Pre-treatment Scores on
"I Think Father Uses Mathematics on the Job"
For the Groups of Sample One and Sample Two

Sample:	Group A	Group B	Group C	Group D	Group E	Group F	Group G	Group H
1 (N=113)	4.38 (N=8)	4.50 (N=3)	4.64 (N=11)	4.49 (N=13)	4.23 (N=55)	4.36 (N=8)	1.67 (N=11)	4.25 (N=4)
2 (N=113)	4.67 (N=9)	3.50 (N=10)	4.57 (N=14)	4.75 (N=40)	3.93 (N=14)	4.89 (N=9)	2.75 (N=8)	4.33 (N=9)

The scores which have caused this difference are clearly contained in the group labeled G in both samples. Obviously we will have to exercise much caution in interpreting these two groups. However the analysis of the mean profile vectors for each group (which is discussed in the next section)

indicates that the G-groups were formed because of the sizes of the change on this scale, and that the scale had minimum effect in determining the formation of all other groups.

Interpretations for Sample One

The eight groups of each sample are formed on a purely statistical basis. The ultimate success of such a procedure depends upon whether or not any meaning can be attached to these statistical groups. The problem encountered in interpreting differences in the mean profile vectors of the groups is one of too much information rather than too little. It is helpful to compare the profile vector components (which are changes between pre- and post-treatment test scores) with mean changes and standard deviations for the same test scores computed for the total population. These means and standard deviations are shown in Table 3. (Means for the two samples are included in Appendix III.)

Table 3: Mean Changes and Standard Deviations of Profile Components for the Total Population

Profile Vector Component	Mean (Change)	Standard Deviation (Change)
Math vs. Non-Math	-0.31	3.40
Math Fun vs. Dull	-0.63	3.06
Ideal Math Self-Concept	-1.62	5.13
Actual Math Self-Concept	-0.56	4.88
I Think Father Uses Math	+0.08	0.71
I Would Like to Take More Math	+0.07	0.79

Table 4 shows the mean profile vectors for each of the eight groups of sample one. The labeling of groups was chosen to emphasize the most extreme groups, but is otherwise arbitrary. Those components whose absolute value is greater than one standard deviation of the change scores for the total population are marked with an asterisk.

Table 4: Mean Profile Vectors for the Eight Groups of Sample One and Relative Group Sizes

Profile Vector Components	Group A	Group B	Group C	Group D	Group E	Group F	Group G	Group H
Math vs. Non-Math	+3.00	+4.75*	-4.00*	-0.54	-1.69	0.00	-3.33	-3.50*
Math Fun vs. Dull	+2.63	+2.73	-5.00*	-0.05	-0.92	-1.64	-1.33	-7.50*
Ideal Math Self-Concept	-1.50	-1.38	+1.00	+0.20	-1.39	-9.73*	-3.00	+3.25
Acutal Math Self-Concept	+5.50*	-5.50*	+4.91*	-1.11	-0.08	0.00	-1.00	-10.50*
I Think Father Uses Math	0.00	+0.37	+0.18	+0.07	+0.10	-0.18	+3.33*	0.00
I Would Like To Take More Math	+1.12*	-0.37	-0.18	+0.05	-0.85*	-0.18	0.00	+0.75
Group N	8	3	11	13	55	8	11	4
% of Sample	7%	3%	10%	11%	49%	7%	10%	3%

*Greater than ± 1 standard deviation of the change scores computed for the total population.

The selection of this level is also arbitrary, but it allows us to focus on the most important components of each group. With this restriction we can make the following observations for the groups of sample one.

Group A: This group has favorably shifted its attitudes toward mathematics during the course of the MTS unit. The high level of change in "Actual Math Self-Concept" can be interpreted as an indication that they feel they are doing better in their mathematics class. The smaller drop in "Ideal Math Self-Concept" complements this interpretation. If the student feels he is actually doing better in mathematics, then his wish to do better than he is should decline slightly as it does here. Finally there is a large upswing in the desire of this group to take more mathematics in the future. Unfortunately, this group is comprised of only 7% of the students in sample one.

Group B: A large increase on the "Mathematics vs. Non-Mathematics" scale characterizes this group. However there is also an unusually large drop on the "Actual Math Self-Concept" scale. Thus while the relative position of mathematics to other subjects has increased during the course of the MTS unit, these students feel that they are doing less well in mathematics. Perhaps

they feel that mathematics is more interesting but also more difficult. This is one of the smallest groups in sample one, comprising only 3% of the sample.

Group C: This group shows unfavorable changes on the "Mathematics vs. Non-Mathematics" and "Mathematics Fun vs. Dull" scales. It would appear that these students find mathematics much less interesting during the course of the MTS unit. However there is a favorable change on the "Actual Math Self-Concept" scale, which can be interpreted as an indication that these students feel they are doing better in mathematics. It is tempting to speculate that they find the work much easier and therefore less interesting. But it is also possible that the actual self-concept changes because they feel they are more satisfactorily working to the best of their ability without reference to the basic interest or difficulty of the mathematics. This group is about 10% of the sample.

Group D: The most interesting thing about this group is the lack of relatively large attitude changes. They make up 11% of the sample.

Group E: This group seems fairly similar to Group D. A slightly unfavorable shift on all attitudes culminates in a sizeable decrease in desire to continue mathematics study. The most striking thing about this group is its size, accounting for nearly half of the total sample.

Group F: The "Ideal Math Self-Concept" scale was designed to measure how a child wishes he were in relation to mathematics. The large negative change on this scale is the most striking feature of this group, and seems to indicate a reduction in the desire to improve performance in the mathematics class. There has been no average change on the "Actual Math Self-Concept" scale however, which would seem to indicate that on the average these students have not perceived a difference in their actual performance in the mathematics class. This group comprises only 7% of the sample.

Group G: The unusually large change shown by Group G on the "I Think Father Uses Math on the Job" scale is not comparable to changes on this scale of other groups, since pre-treatment scores for Group G were unusually low. Since the change on this component is the only one exceeding the standard deviation of the change distribution for the total population, it seems reasonable to conclude that this component has been primarily responsible for the formulation of this group. Computation of the sum of the squares of the differences of the other five components for the other groups compared to G, shows that Group G is closest to Group E, already the dominant group of this sample.

Group H: This group is characterized by large unfavorable changes on three of the attitude scales. The relative position of mathematics to other school subjects has decreased. Mathematics is seen as much duller. Students perceive that they are not performing as well in mathematics, and the increase in the score on the "Ideal Math Self-Concept" scale indicates that they wish they were doing better. Paradoxically, there is a rise on the "I Would Like to Take More Mathematics" scale. The attitude changes of this group are so unfavorable that we wonder if the intention of these students was "I Would Like to Take Some Different Mathematics!" The group is small, accounting for only 3% of the sample.

Interpretations for Sample Two

Hierarchical Grouping Analysis is not predictive; there is no guarantee that sample two will match sample one at all. In detail, it does not, as Table 5 shows. As before, the labeling of groups was arbitrarily chosen to emphasize differences between groups and similarities with sample one.

Table 5: Mean Profile Vectors for the Eight Groups of Sample Two and Relative Group Sizes

Profile Vector Components	Group A	Group B	Group C	Group D	Group E	Group F	Group G	Group H
Math vs. Non-Math	+2.78	+3.70*	+1.03	-1.53	-1.72	+1.00	-1.62	-5.67
Math Fun vs. Dull	+4.33*	-1.30	-1.07	+0.08	-0.93	-0.55	-0.12	-7.66*
Ideal Math Self-Concept	-2.34	+1.50	-5.86*	+0.13	-2.07	-8.89*	-4.12	-0.89
Actual Math Self-Concept	+4.22	-5.50*	+5.64*	-1.62	+2.22	-3.89	+1.00	-5.45*
I Think Father Uses Math	-0.34	+0.60	+0.07	-0.15	+0.14	-0.66	+1.25*	0.00
I Would Like To Take More Math	+1.00*	+0.10	0.00	+0.05	-1.07*	-0.11	0.00	-0.44
Group N	9	10	14	40	14	9	8	9
% of Sample	8%	9%	12%	36%	12%	8%	7%	8%

*Greater than ± 1 Standard Deviation of the change scores computed for the total population.

The following interpretations, which are based upon the major changes of the components of the profile vector, do show similarities to the groups of the previous sample. The eight groups of sample two may be described in the following ways.

Group A: A large increase on the "Mathematics Fun vs. Dull" seems to indicate that this group sees mathematics as being more fun during the course of the MTS unit. There is an increase in the "Actual Math Self-Concept" scale accompanied by a slight decrease in the "Ideal Math Self-Concept" scale. As discussed previously, it is possible to interpret these changes as complementary. There is an increase in the desire to take more mathematics. But this group, whose attitudes toward mathematics have changed favorably, makes up only 8% of sample two.

Group B: This group has favorably shifted its attitudes toward mathematics with respect to other school subjects. However, there is also a large decrease on the "Actual Math Self-Concept" scale. Apparently these students feel that their performance in the mathematics classroom has been less satisfactory during the MTS unit. The group comprises 9% of the sample.

Group C: A large increase on the "Actual Math Self-Concept" scale indicates that this group feels that their performance has improved during the MTS unit. A complementary decrease in the wish to be doing better is indicated by the decrease on the "Ideal Math Self-Concept" scale. The slight decrease on the "Mathematics Fun vs. Dull" scale shows a smaller tendency to view mathematics as somewhat duller. On the other hand the "Math vs. Non-Math" scale shows that mathematics has increased in favor in relation to other subjects. Perhaps there is more than a trace of ambiguity in the attitude changes of this group. The group accounts for 12% of the sample.

Group D: The relative size of this group (36% of the sample) makes it important. However it seems to be characterized by lack of large attitude changes. There is some tendency to make less favorable comparisons between mathematics and other subjects, and to see actual classroom performance in mathematics as less satisfactory.

Group E: This group shows a decrease in desire to take more mathematics courses in the future. However they perceive that their performance in mathematics has become slightly more satisfactory and this is complemented by a slight decrease on the "Ideal Math Self-Concept" scale. This group is 12% of the sample.

Group F: The most striking feature of this group is the large decrease on the "Ideal Math Self-Concept" scale. There is less desire on the part of the student to improve his performance in class after the MTS unit than before. Furthermore there is a slight decrease in the way he views his actual performance in the mathematics classroom. This group comprises 8% of the sample.

Group G: As with sample one, the large change on the "I Think Father Uses Math on the Job" scale which characterizes Group G is not comparable to other groups because of pre-treatment score differences. Again it seems reasonable to conclude that this component has been primarily responsible for the formation of this group. Computation of the sum of the squares of the differences of the other five components for the other groups compared to G, shows that Group G is closest to Group E. This agrees with the finding for sample one.

Group H: Large unfavorable changes on three attitude scales characterize this group. Attitude toward mathematics has become less favorable in comparison to attitudes toward other school subjects. Mathematics is seen as duller. Students' perceptions of themselves in the mathematics classroom have changed unfavorably. There is some decrease on the "I Would Like to Take More Mathematics" scale. In general the attitude changes of this group during the MTS unit have been unfavorable. The group comprises 8% of the sample.

Comparisons of the Two Samples

The order in which the groups are reported by the grouping program depends ultimately on only the data for the first individual considered, since the vector formed with this data is considered as the first "group." We have ordered the groups in Tables 4 and 5 to emphasize their similarities and differences. The following observations and cautions may provide a helpful overview and comparison of the two samples. (We will proceed in our discussion from groups that are obviously similar to groups where the similarity is less clear.)

A-Groups: Differences between these groups seem to be differences of degree rather than kind. The groups are clearly similar, both in relative size and in their favorable attitude changes toward mathematics.

B-Groups: There are striking similarities between scores on the "Math vs. Non-Math" scales as well as on the "Actual Math Self-Concept" scale. However, there are discrepancies on the "Mathematics Fun vs. Dull" and "Ideal Math Self-Concept" scales. The latter may only be a minor discrepancy both because of relative sizes and possible alternate interpretations of the

meaning of a decrease on this scale. The former discrepancy does appear to be more serious, however, because of the size of the positive change in sample one. These groups now see mathematics as more interesting; but feel they do less well at it.

H-Groups: There is a major disagreement on the "Ideal Math Self-Concept" scale for these two groups. The "I Would Like To Take More Mathematics" scale also presents a discrepancy. Nevertheless, the similarities on the remaining four scales are so striking that it seems reasonable to consider the groups as similar in their unfavorable attitude changes toward mathematics.

D-Groups: These groups seem clearly similar. The discrepancies on the "Mathematics Fun vs. Dull" and "I Think Father Uses Mathematics on the Job" scales are not serious, since the change scores for these scales are so close to zero. These groups seem best characterized by the lack of large changes.

E-Groups: These groups also seem very similar. There is a discrepancy on the "Actual Math Self-Concept" scale, but except for this the differences are mainly of degree. These groups seem very similar to the D-Groups except for their larger negative changes on the "I Would Like to Take More Mathematics" scale. The fact that the E-Group is the largest group in sample one while the D-Group is the largest in sample two would tend to support their similarity.

G-Groups: Except for degree, these groups differ only on the "Actual Math Self-Concept" scale. Since the scores on this scale are less than one-fourth of a standard deviation, it is probably acceptable to ignore their effect on the profile vector. Since the changes on the "I Think Father Uses Math on the Job" scale are unreliable these groups should be considered subsets of the E-Groups.

F-Groups: The major differences in these groups seem to occur because the mean change scores for the "Math vs. Non-Math" and "Actual Math Self-Concept" scales are zero for sample one. The other four scales seem to agree quite well, differing only in degree. These groups are characterized by the sharp drop on the "Ideal Math Self-Concept" scale.

C-Groups: These groups are probably the least similar of any of our pairings. Major discrepancies exist on the "Math vs. Non-Math" scale and the "Ideal Math Self-Concept Scale." The remaining scales seem similar although there is difference in the size of the change scores on the "Math Fun vs. Dull" scale. It appears that this group perceives that their class performance is better after the MTS unit but at the same time feel that mathematics is duller.

To summarize, Groups D and E-G account for most of the students in our samples and show very little attitude change. The H-Group appears to be basically unfavorable in its attitude changes, but accounts for only about 6% of the combined samples. The A-Groups undergo strongly favorable changes in attitude, and account for some 8% of the combined samples. Groups B and C appear to be opposites. The B-Groups feel that mathematics is more interesting, but feel they do less well. The C-Groups feel that mathematics is less interesting, but feel they do better. The B-Groups account for about 6% of the combined samples; the C-Group accounts for approximately 12%. The F-Groups change primarily in the "Ideal Math Self-Concept" scale. How they wish they were in relation to mathematics becomes less favorable. They account for about 8% of the combined samples.

Sex and Teacher Composition of Groups

Tables 6 and 7 show the numbers of girls and boys comprising each group of samples one and two, respectively. The number of teachers (classes) represented in each group is also included.

Table 6: Sex and Teachers Composition of Sample One Groups

	Group A	Group B	Group C	Group D	Group E	Group F	Group G	Group H
No. of girls, N=67	4	1	7	10	31	7	5	2
No. of boys, N=46	4	2	4	3	24	1	6	2
Teachers represented, N=29	7	3	10	10	27	8	9	4

Table 7: Sex and Teacher Composition of Sample Two Groups

	Group A	Group B	Group C	Group D	Group E	Group F	Group G	Group H
No. of girls, N=60	6	3	6	20	10	4	6	5
No. of boys, N=53	3	7	8	20	4	5	2	4
Teachers represented, N=29	7	9	11	24	11	9	7	8

Boys tend to dominat the B-Groups of both samples. The domination of Groups D and F by girls in sample one is not confirmed by sample two. The fact that there are nearly as many teachers represented as students in each group when the group size is less than twenty-nine indicates that no single teacher has been inordinately responsible for the placement of students in a particular group.

Analysis of Non-Profile Attitude Scales

How do our eight "natural" attitude groups compare on the attitude scales which were not used in the grouping procedure? This question was investigated by repeating univariate analysis for these twelve scales for first post-treatment and then pre-treatment measures. These statistical tests are included in Appendix IV. Only the F-ratio for the post-treatment scores on the "Debilitating Anxiety" scale for sample one exceeded the critical value of 3.14 for the $p < .01$ level (7 and 105 degrees of freedom). The mean score on this scale for each group is shown in Table 8. Sample two is included for comparison.

Table 8: Mean Post-Treatment
Debilitating Anxiety Scores by Group

	Group A	Group B	Group C	Group D	Group E	Group F	Group G	Group H
Sample one	23.75	33.37	30.55	27.05	30.23	23.73	32.00	35.25
Sample two	32.67	26.80	27.71	28.40	30.00	27.89	26.25	32.22

The total mean score for sample one is 28.07. It would appear that most of the difference is attributable to the high post-treatment level of debilitating anxiety shown by Group H (a group that generally showed unfavorable attitude changes). At the same time Groups A and F show low levels of debilitating anxiety. (It will be recalled that Group A showed generally favorable attitude changes while Group F changed unfavorably on the "Ideal Math Self-Concept" scale.)

Sample two fails to confirm the differences shown by the group of sample one. Here the difference in mean scores for post-treatment debilitating anxiety is much smaller between Groups H and F, and practically nonexistent between Groups H and A. Thus the significant difference in post-treatment debilitating anxiety appears to be a function of the clustering trends of the scores of the particular individuals which made up sample one. We cannot predict levels of debilitating anxiety for other groups based on this information alone.

Achievement and Ability of Groups

How do our eight "natural" attitude groups compare on ability and achievement scales? If interest in attitude changes stems from the effect of attitude upon achievement, then the ultimate significance of the eight groupings must lie within the answer to this question. The analyses of variance reported in Appendix IV show that the differences in attitude

patterns among groups are <u>not</u> reflected in significant differences in either ability or achievement.

The "Necessary Arithmetic Operations" scale was given during the pre-treatment testing as a quick initial estimate of ability. Table 9 shows the mean scores on this scale for the eight groups in each sample. It is interesting to note that the favorable A-Groups do not score highest on this scale; nor do the unfavorable H-Groups score lowest. The C-Groups whose change on the "Actual Math Self-Concept" scale was favorable do show low scores. However the most interesting thing about the table is the apparent lack of pattern between scale scores and groupings. This lack is confirmed by a univariate analysis of variance on both samples for this scale (test 22). The F-ratios for samples one and two are 0.39 and 1.99 respectively for 7 and 105 degrees of freedom; a hypothesis of no significant difference between groups cannot be rejected for $p < .05$ (critical F-ratio value, 2.42).

Table 9: Mean Necessary Arithmetical Operation Scores for the Eight Groups

	Group A	Group B	Group C	Group D	Group E	Group F	Group G	Group H
Sample one	7.87	7.12	6.73	7.25	7.08	8.00	7.00	7.25
Sample two	8.00	8.80	6.50	6.70	6.79	7.11	5.63	7.56

Analyses of variance by groups are found for both pre- and post-treatment administrations of the three algebra scales in Appendix IV. The three pre-treatment scores are tests 24, 25, and 26. F-ratios do not reach the critical value for $p < .05$ for any of these tests, so that hypotheses of no significant difference between groups cannot be rejected for pre-treatment algebra scales.

Mean changes in algebra scale scores for groups of both samples are shown in Table 10.

Table 10: Mean Changes in Algebra Scale Scores
Between Pre- and Post-Treatment

Sample One:	A	B	C	D	E	F	G	H
Algebraic No. Properties	-.13	+.25	+.83	+.38	.00	+1.00	-.33	-.25
Algebra Sentences	+.99	+.75	+.36	+.49	+.54	+.82	+.33	-1.25
Algebra Translation	+.13	+.49	.00	+.47	-.07	+.82	+.33	+.50

Sample Two:	A	B	C	D	E	F	G	H
Algebraic No. Properties	+.44	+.30	+.72	+.20	-.15	+.45	+.30	+1.33
Algebra Sentences	+.35	+.70	+.93	+.30	-.07	+.22	+.75	-0.11
Algebra Translation	.00	+.30	+.29	+.25	+.29	+.66	-.25	+.89

As with ability scale, the most remarkable thing about Table 10 is the lack of pattern it shows between groups and between samples. This appears to be borne out by an analysis of post-treatment scores since no F-ratios approach the critical value except for the post-treatment "Algebra Number Properties" scale for sample one, where the F-ratio value is 3.71 (significant at $p < .01$). The mean score on this scale for each group is shown in Table 11. Sample two is included for comparison.

Table 11: Mean Post-Treatment Algebraic
Number Property Scores by Groups

	Group A	Group B	Group C	Group D	Group E	Group F	Group G	Group H
Sample 1	3.62	3.50	3.82	3.73	3.85	4.91	1.67	1.75
Sample 2	4.00	3.90	3.79	3.27	3.21	3.89	4.00	3.44

The spread between Groups F and H has increased during the treatment interval for the sample one groups. Students have previously studied number properties from their standard texts. There is no direct work on number properties in the MTS unit, and the work which it does contain on algebraic expressions would seem to apply only indirectly to this scale. Thus it may be that the number properties scale acts as an achievement scale for some students and as a retention scale for others.

Whatever the interpretation, the post-treatment differences in algebra number properties are not common to both samples and seem to be a function of specific sample clusterings. We must still conclude that attitude change

clusterings are not accompanied in any general way be differences in achievement.

(A final glance at Appendix IV also shows critical F-ratio values for $p < .01$ for post-treatment scores on the attitude scales "Ideal Math Self-Concept" for sample one and "Math Fun vs. Dull" for sample two. However both these scales are involved in the profile vectors used for the groupings procedure, and Veldman points out that as such they are "artifically and unreliably significant" (Veldman, p. 311).)

Final Interpretations

With the exception of one case of the "Algebra Number Properties" scale, we have failed to find evidence which would allow us to reject hypotheses of no significant difference in achievement scores among groups. The natural clusterings of attitudes which occurred during the time period of the MTS unit were not accompanied by similar clusterings in achievement. This finding seems to agree with the results of other attitude-achievement studies. Thus it appears that attitude-change clusterings are not a major consideration if one is concerned with mathematics achievement during a unit taught via physical approaches.

In fact, the majority of our samples changed their attitudes toward mathematics very little during the course of this study. Thus it would seem that, in general, strong cohesive attitude changes among students are not a factor for major consideration in developing or adopting mathematics taught via physical materials at the middle grades.

Overall significant attitude changes in our population seem to be due to three or four relatively small natural groups for each sample. The most negative of these was the group designated "H" in both samples. Their attitude changes do seem strong, cohesive and unfavorable in comparison to the rest of the samples. But, they comprise only 6 percent of our combined samples; only about half of the expected size if the groups were filled at random.

At the other end of the clusters is about 8 percent of the combined samples which has changed its attitude favorably toward mathematics. They express a greater desire to take more mathematics at the conclusion of the study. They see mathematics as more fun, and their actual classroom performance as better at the end of the MTS unit. In view of the advanced concepts introduced in their mathematics class by the MTS unit, it may be remarkable that these changes occurred at all. Nevertheless it seems

reasonably safe to view this group as students who liked almost all aspects of the MTS unit.

We cannot make such a simple interpretation for the remaining groups. Their attitudes lead one to guess that they liked some aspects of the MTS unit while disapproving of others. The groups designated "B" in each sample are a case in point. These groups improved their favorable attitude toward mathematics in comparison to other school subjects, but at the same time seemed to acknowledge a slightly poorer classroom performance via a decline in score on the "Actual Math Self-Concept" scale. One possible interpretation of this ambiguity is that they found the MTS unit comparatively interesting but also more difficult. These groups are comprised primarily of boys which tends to strengthen an interpretation of increased interest in physical materials.

C-Groups can be interpreted as logical opposites of B-Groups. In particular these groups increase their actual math self-concept, which can be interpreted as a feeling that they are performing more satisfactorily in the mathematics classroom. At the same time a decline on the "Math Fun vs. Dull" scale would seem to indicate that mathematics is seen as more boring. One possible interpretation is that this group sees the activities of the MTS unit as accessible but at the same time, less interesting.

The remaining group-type (F) is the most difficult to explain. These groups are characterized by large drops on the "Ideal Math Self-Concept" scale. This means that at the end of the study they are less likely to agree with statements like "I wish it were easier for me to talk in front of my mathematics class," and "I wish I were more proud of my mathematics homework." It seems reasonable that students could make this change for a variety of reasons which all culminate in a decrease in concern about their performance in mathematics. Perhaps they are so uninterested in the MTS unit that they no longer care. Perhaps they feel so limited in ability as a result of the new content that they no longer find it even remotely realistic to desire to be performing better. On the other hand they could be satisfied that their classroom performance is adequate for the new unit. None of these reasons are supported by changes on other profile scales. Perhaps this is a group-type which has changed attitudes for a wide variety of reasons.

What happened to attitudes during the study of the MTS unit? Most students changed attitudes very little. Some liked the unit, some liked it but found it harder; others found it easier but less interesting, and a few

disliked it quite strongly. In no case were these groups large enough to represent major factions.

To be sure, attitudes did decrease significantly on the six scales used to form the profile vector. In Appendix V we compare the attitude shifts of our student population to those of the NLSMA Y-population measured at the 7th, 9th and 11th grades. Population comparison can be very risky. The fact that the pre-treatment measurement of attitudes for our population in spring of grade eight yields mean scores generally lower than those of the Y-population measured in the fall of grade nine should alert one to the fact that the student population for this study cannot be considered to be a random sample of the Y-population. Nevertheless, the decrease in attitude scores for our population seems compatible with the trend for the NLSMA population.

What do all these attitude-changes mean? Perhaps it is not entirely facetious to compare an eighth grader's preference to various presentation-styles of mathematics to a preference for various types of poison. If the end effect is the same, the method of arrival is of only minor concern. Before accepting this conclusion too readily, however, we should realize that it is entirely possible that the attitude scales used in this study could tend to bias our results in this direction. Our opinion statements do not ask about the kinds of interactions which occur (or do not occur) in the mathematics classroom. Only 7 of 87 items mention "mathematics class." This omission is not necessarily an oversight. We believe it tends to arise from a covert assumption that interest in mathematics and in the way it is communicated are the same things. The written responses of students on an Evaluation of Materials questionnaire leads us to suspect that this assumption may not be true. Many students appeared to be most enthusiastic about being given the opportunity to actively participate in their mathematics class, but something less than enthusiastic about the content material they were to master as a result of this activity. Thus we wonder if there may be a significant difference between the agreement that would be obtained by the statement "Mathematics is fun" (which we used) and the statement "Mathematics class is fun."

This problem is worthy of further study. The production of uniformly favorable attitudes toward mathematics seems neither reasonable nor desirable. But the production of favorable attitudes toward learning does appear to be an important goal. We should develop a variety of mathematics communication processes which contribute to this goal. This study seems to indicate that this development is not an easy matter.

Finally, the reader should be aware, as we are, that we have been engaged in "the great game of averages." We have formed statistical groups and described them with statistical means. Just as it is possible that no student actually exists whose attitude scores are the same as the means we have been working with, so it may be that the groups we have formed do not manifest themselves in overt behaviors which would make them apparent to a classroom observer. Arithmetic means are a simple, yet powerful, tool for reducing the complexity of a group to a comprehensible simplification; yet they do not imply that this complexity never existed. Much the same thing can be said for the procedure we used to form our groups. It too provides simplification; but it does not imply that the complexity of individuals never existed. In fact our ultimate failure to find a few strong cohesive attitude groups can be viewed as essentially a back-handed tribute to the individuality of the classroom student. For those who seek in the MTS approach a unitary curriculum structure for all students, he remains an unyielding barrier.

Appendix I

POPULATION DESCRIPTION

Table I: Schools, Teachers and Classes

School	Teacher	Sex	Class hour	Class length (min)	Student N	Text	Chapter before MTS Unit
L. C. Curtis	A	M	10:30	45	32	Expl Mod Math	Ch. 4
	B	M	8:45	45	27	Mathematics 8	Ch. 8
	C	M	11:30	44	34	Expl Mod Math	Ch. 4
	D	M	9:00	45	33	Expl Mod Math	Ch. 4
Jefferson	A	F	9:45	42	29	Mathematics 8	Ch. 7
	B	M	1:00	--	33	Mathematics 8	Ch. 7
Mango	A	F	8:15	48	32	Mathematics 8	Ch. 7 + 12
	B	M	8:15	48	33	Mathematics 8	Ch. 7 + 12
	C	F	8:15	48	31	Mathematics 8	Ch. 7 + 12
	D	M	8:15	48	32	Mathematics 8	Ch. 7 + 12
Juan Cabrillo	A	M	1:45	--	36	Mathematics 8	Ch. 12
	B	M	9:30	45	30	Mathematics 8	Ch. 7
	C	M	11:15	--	36	Expl Mod Math	Ch. 5
	D	M	1:45	53	34	----------	
Ortega	A	M	9:15	--	30	Mathematics 8	Ch. 12
Patrick Henry	A	M	2:30	45	32	Expl Mod Math	Ch. 5
	B	M	11:30	45	34	Expl Mod Math	Ch. 5
Raymond J. Fisher	A	M	8:30	50	36	Mathematics 8	Ch. 9
	B	M	11:15	45	30	Mathematics 8	Ch. 9
	C	M	1:30	45	33	Mathematics 8	----------
	D	M	10:30	45	34	Mathematics 8	Ch. 9
	E	M	8:30	45	34	Expl Mod Math	Ch. 6
Rogers	A	M	12:00	40	35	Mathematics 8	Ch. 7
	B	M	8:30	45	33	Mathematics 8	----------
	C	M	2:00	45	35	Mathematics 8	Ch. 8
	D	F	1:45	45	34	Expl Mod Math	Ch. 5
	E	M	9:00	49	36	Mathematics 8	Ch. 12
Warren E. Hyde	A	M	-----	--	28	Mathematics 8	----------
	B	M	1:30	--	31	Understanding Arithmetic	

The next table ists the major topics covered by the two most-used texts: Mathematics 8 and Exploring Modern Mathematics through the first twelve and six chapters respectively. The most notable differences relevant to this study are in the areas of graphing and the use of exponents (scientific notation). The brief section on the law of the lever (Exploring Modern Mathematics, Chapter 6) does share common ideas with the first chapter of the MTS unit; however only one teacher reported that her class had covered this material prior to the study.

Table II: Content Covered By Students Prior to MTS Unit

Text: Exploring Modern Mathematics

Number Properties
- integers (positive and negative)
- absolute value
- addition and subtraction on the number line
- simplifying phrases
- conditional sentences
- graphs on a plane
- graphs of number sentences (integer-values only)

Congruence
- constructing angles
- bisecting angles
- bisecting segments
- constructing perpendiculars
- parallel lines and transversals
- angles formed by transversals
- quadrilaterals (and perimeters)
- circles (and circumferences)

Exponents
- Mult. and division using exponential notation
- negative integers as exponents
- scientific notation
- rational numbers
- order and the number line
- operations with rational numbers
- square roots

Exploring Modern Mathematics (continued)

Graphs on a Plane
areas of rectangles
generating area formulas for trapezoids, triangles, parallelograms and circles using sweeps pythagorean property of right triangles
circle graphs

The law of the Lever
factoring integers
equivalent phrases
equivalent fractions
long division with rationals

Text: Mathematics 8

Numeration Systems
various bases
set notation and 1 to 1 correspondence
properties of natural numbers
number line and negative integers
order relations
properties of equations
solving problems with simple equations
factors, primes and divisibility
exponents
rational numbers and operations with

Points, lines, planes, intersections
parallel lines and planes
polygons
angles and angle measurement
perpendicular lines and planes

Measurement, precision and accuracy
constructing congruent segments and angles
bisecting line segments and angles
constructing perpendiculars
constructing congruent triangles
properties of congruent triangles
parallel lines and corresponding angles
proving triangles congruent

Mathematics 8 (continued)

Perimeter and circumference

area of rectangles and triangles

areas of parallelograms, trapezoids and circles

prisms, pyramids and cylinders

volumes of cylinders and cones

spheres

Ratio

proportion

percents

Appendix II

EXPERIMENT MODIFICATIONS AND SUPPLEMENTARY TEACHER'S NOTES

MATHEMATICS THROUGH SCIENCE

Experiment Modifications

1.2 The Seesaw Experiment

One box of jumbo paper clips (2" length) was substituted for the set of standard weights. Each clip was used as one weight unit. The paper clip hangers specified in the text can then be counted as one unit instead of ignoring its effect (as the text implies).

The support stand fashioned from Dixie cups was omitted. Instead the C-clamp specified for experiment 2.2 was used to clamp the triangular file directly to the desk top. Several teachers discovered that even this could be simplified by hanging the paper clamp directly on the handle of the C-clamp.

1.12 Finding Unknown Masses by Experiment

All modifications for experiment 1.2 were also employed in this experiment. The unknown weight was hung from a light thread instead of the paper clip hanger specified in the text. (If the paper clip hanger is used, it must be counted as one unit added to the unknown. The resulting equation gives interesting practice in handling grouping symbols, but was judged to be inappropriate at this point in the course.) Many teachers weighed the paper clips on standard scales to obtain the weight of the unknown in standard units.

2.2 The Loaded Beam Experiment

One 12" plastic ruler was substituted for the specified 15" wooden ruler. Again jumbo paper clips were used instead of standard weights. Since a paper clip causes only a small deflection of the ruler, students were instructed to add five of them at a time. Some teachers found that the ruler could be taped to the desk top with several strips of masking tape instead of using the C-clamp--this is possible because of the smaller weight of the paper clips.

2.10 The Falling Sphere

Magnets were omitted from this experiment. Each class was supplied with 100 steel ball bearings. A new ball bearing was dropped for each trial. At the end of the class, the teacher retrieved the balls with a large magnet borrowed from the science department. Frosty cellophane tape, which can be written on directly, was used instead of paper strips.

Timing was done by recording the busy signal from a telephone on the school's tape recorder. In practice, the time interval between beeps is a little short for this experiment; taking two such intervals as a unit (counting every other beep) gives better results.

3.2 The Trampoline Experiment

This experiment was drastically simplified although the intent of the experiment was not modified. Instead of bouncing marbles on a rubber "trampoline," ping-pong balls were bounced on the floor. Paper was taped to the classroom wall so the approximate bounce heights could be measured directly for up to five bounce returns. The ball was always dropped from a height of one meter by releasing it from the hand held at the top of a meter stick. Students did not repeat the experiment for other types of balls as the text suggests, but this could have been done using rubber balls or superballs. These modifications allowed students to do the experiment in groups of two instead of observing a teacher demonstration.

3.6 Gay-Lussac's Law Experiment

This experiment was done as suggested, except that the Gay-Lussac apparatus was fabricated instead of being purchased directly. To make the apparatus, a hole was drilled in the rod-socket of a brass toilet tank float, and a one foot length of copper tubing was soldered to the socket. Copper tube fittings were attached to the other end of the tube to allow it to be threaded onto a #94030 Cenco vacuum gauge. In use, the brass float was heated in boiling water before attaching the vacuum gauge; as the air cooled, the gauge registered the loss in air pressure.

MATHEMATICS THROUGH SCIENCE

Supplementary Teacher's Notes

The traditional way to conduct a laboratory class is to tell all about the experiment first -- what it is, how it works, how to do it, and what to expect to find. The trouble with this is that if the experiment is thoroughly explained there is no reason for the student to do it (except to check up on the teacher). Obviously, some minimal instructions must be given initially, but for an effective discovery lesson, a good rule to remember is

DO FIRST --- EXPLAIN LATER!

Hopefully, the students will explain to you what they have understood from the experiment.

You should be aware that the text materials do not completely adhere to this philosophy. Therefore, as far as students are concerned, it may be wise to change the rule to "do first -- read later." The text sections which explain the experiments can almost always be treated as a follow-up to an experiment, as a review, or summary.

Perhaps a word about the objectives of the experiments is in order. For our course, the aim is to find out how the experimental objects behave, not why. How calls for describing conditions and resulting behavior, and one of the best ways to do this is by using mathematics. We want to concentrate on learning the mathematical ideas which are most useful in describing how.

By contrast, explaining why certain objects behave the way they do is a much less precise and more complicated matter. Why usually involves building scientific models or theory. While this is interesting in its own right, it does not propertly reflect the purpose of this unit. The theory which attempts to explain why a meter stick balances, for example, involves such complex ideas as vectors, vector products, torques, center of mass, etc. Don't let your students drag you off into these murky waters! Our purpose is much simpler: to describe as precisely as we can how to balance a meter stick. The quest for precision leads us to think about number sentences and number phrases -- from science to a mathematical idea.

Finally, try to help the students avoid the notion that the experiment gave them the "wrong answers." Nature cannot behave incorrectly. Perhaps they get the wrong data becuase they cannot read a meter stick correctly. Or perhaps they get the right data, but data from the "wrong experiment." For example, you may have some groups who will try to explain mathematically how

to balance a meter stick that is so twisted around the file that it cannot swing freely. Of course, their data will not match the rest of the class; not because they got the wrong answers, but because they asked the wrong questions. Try letting the class look at everyone's data to try to discover relations, and to decide for themselves which data should be eliminated as unreliable. (If you get a conscientious class that wants to eliminate all the data, don't despair! Let them do the experiment over again if they feel they can improve. If you have a class of students that is that involved and interested, pat yourself on the back!)

HINTS FOR CHAPTER ONE

1.2 Experiment

1. To eliminate effects from the meter stick, you must be sure that it will balance at the 50 cm mark. Put the file on the desk, and lay the meter stick flat across the edge on the 50 cm line. If it doesn't balance, tape some small washers on the lighter end to bring it to balance.

2. Use the C-clamp to clamp the file to the desk top so that the small end of the file (not the handle) sticks over the edge of the desk. (At this writing, not all of the C-clamps had arrived. You should be able to accomplish the same thing by taping the file to the desk top with two or three strips of masking tape.) Put the paper clamp on the meter stick, and hang the balance from the file. Be sure the clamp does not bind on the file, and that the pivot point is over the 50 cm position.

3. Use paper clips for weights, using 1 clip for every 10 grams specified in the text. (This will amount to dividing all the gram measures in the table by 10.) Bend one paper clip into an "S"-shape to use as a hanger -- but be sure to count it as one of your weights!

4. Be sure to distinguish between position where the weights are hung, and the distance to the pivot. The marks on the meter stick indicate the position of the weights. You may have to subtract to find the distance.

5. You can judge more easily when the meter stick is horizontal (and avoid spills!) if you let a heavy textbook protrude from the desk top over one end of the meter stick. Measure (with a ruler, pencil, stick, paper, card, etc.) the distance of the stick below this book before weights are added. Then the weights must be shifted until this same height is achieved again.

6. This process (in #5) is tedious. If the class lacks patience, have them measure to only the nearest centimeter. You can encourage pride in the student's work, however, by having everyone copy their results in a large table on the blackboard to see who gets the "best" results. Try averaging class results.

7. 120 = 121?! Not usually, but under certain conditions of measurement we may be willing to accept such an equality. The meter stick balance gives a good chance to discuss measurement errors. There is a limit (sometimes sizeable) to the balance's sensitivity. For example, if we hang 20 clips 6 cm from the pivot we can balance them with 4 clips 30 cm from the other side of the pivot. But most balances won't show any difference if the 4 clips are hung a little to either side of the 30 cm distance (try it experimentally). You may decide that the balance distance is as much as 30.6 or as little as 29.4 cm. Sometimes this is written as 30 ± 0.6 cm. The product will be as much as 122.4 or as little as 117.6. Thus in our hypothetical case, 120 may be equivalent to anything from 117.6 to 122.4 because our balance cannot tell us the point of balance, but only a range of balance. Although the details are different, this final range of error is a characteristic of all measurements. Keep it in mind with all the work in this unit.

8. What are the physical factors that limit the sensitivity of the meter-stick balance? See how big a list your students can make (some of them may surprise you).

1.3

Assuming the weight "m" is placed on the right-hand side of the meter-stick balance, write a number sentence which will be true if the balance tips down to the right. ($m \times d > 120$) You can use this idea in Section 1.11, also.

1.6 Distributive Property

You can use the meter-stick balance to illustrate this, too. 5 clips at a distance of 40 cm on one side will balance 5 clips at 10 cm (distance) plus another 5 clips at 30 cm (distance), if

$$5(40) \stackrel{?}{=} 5(10) + 5(30)$$

or if

$$5(10 + 30) \stackrel{?}{=} 5(10) + 5(30).$$

1.12 Experiment

Do not hang the unknown object on a paper clip hook as the text suggests, since a paper clip is one of our weight units! Use a loop of light string or thread instead (you can ignore the weight of the thread). Use your imagination in selecting unknown objects (objects with holes in them will be much easier, so try rings, bracelets, scissors, etc.). The weight of the object you obtain will be its weight in paper clips, of course. If your students are less than thrilled with this, they can convert to regular weight units (and get some multiplication practice) by knowing that 1 paper clip weighs about 0.07 ounces or 0.004 lbs.

HINTS FOR CHAPTER TWO

2.1 Experiment

1. Clamp the plastic ruler near the end, so as much extends beyond the desk as possible. (If the C-clamps have still not arrived, tape it down with masking tape.) Clip a paper clip on the far end to make a "loop" to hang the other clip-weights onto. You need not count this as a weight if it is on when the zero-weight reading is taken (we can just pretend that it is a part of the ruler).

2. Add clip-weights in increments of five (at least), so that the weight column in the table reads 5, 10, 15, 20, 25, 30, etc. Even then, students will need to carefully read the meter stick since the change will be only 0.1 or 0.2 cm with each addition.

3. You may want to check your student's scale-reading ability before you begin this experiment. Try a short quiz using a transparent ruler on an overhead projector. Point at several points with a pencil and have students write down the scale reading in each case.

2.2 Graphing the Experimental Points

If you are not sure about your student's ability to graph, try playing the tic-tac-toe game (student teams tell the teacher where to put X's and O's by giving the proper coordinate pairs--the symbols are placed at the vertices of the graph boxes instead of in the centers of the usual game).

2.5 Slope

Have students compute the slope of a line using different pairs of points on the same line. Getting the same value for the slope is where the use of ratio pays off.

We can attach physical meaning to the value of slope, also. Since "rise" corresponds to a change in position or the "bend" of the ruler, and "run" corresponds to change in the number of paper clips, the slope value is an indirect measure of "bend per paper clip." Although the slope of any straight-line graph is calculated in the same way, the physical meaning of its value will be different according to what physical attributes were plotted on what axis. Thus, each axis should be labeled to show what kind of quantities are being plotted. The axes of the graph in the text are labeled. What is "l"? What is "p"?

2.6 Equation of a Straight Line - Slope-Intercept Form

Here is one suggestion for motivating this section so that the "arbitrary mathematics" becomes a little more palatible:

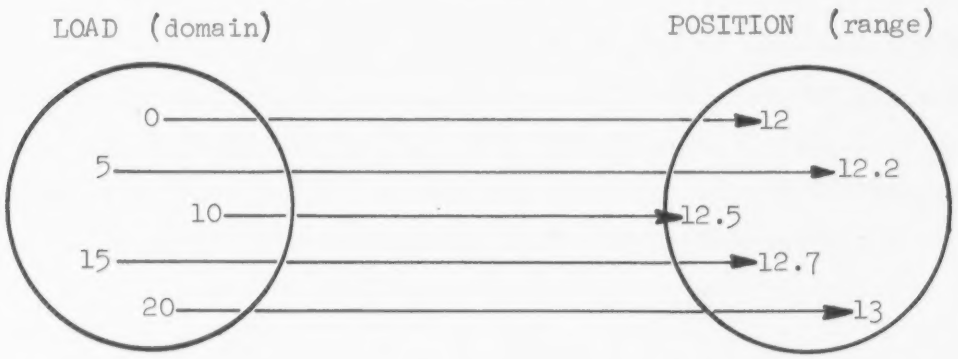
Look at the graph of experimental data (you may want to present a "best-average" graph for class consideration). Who can guess a "computer rule" so that when given the number of paper clips for load, we can compute the position of the ruler? Does it help to know the slope value from the preceding section?

When you do guess the rule, can you write it in equation form?

Is there a faster way to write a graph-line equation than guessing?

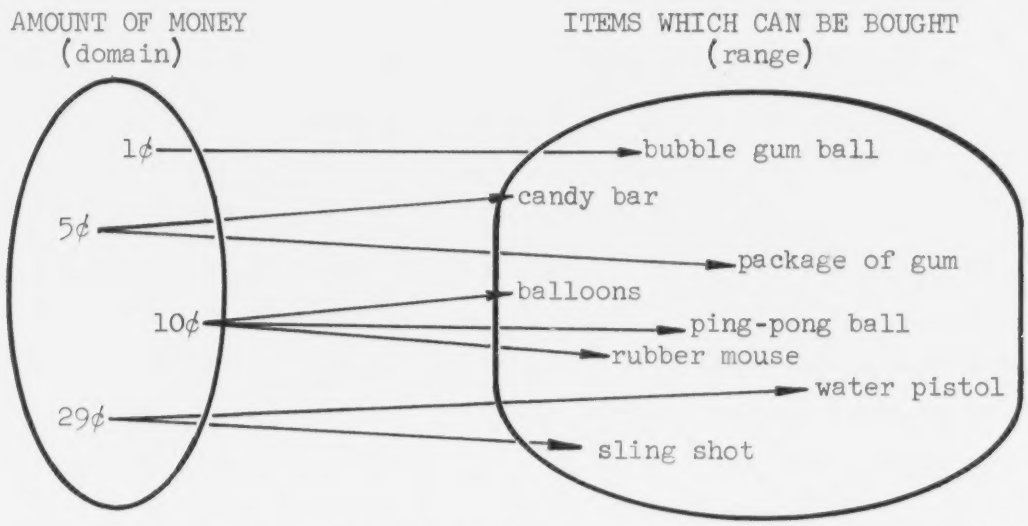
2.8 Relations and Functions

Try using the following Venn diagram to picture the function idea:



What makes this a function (as opposed to a relation) is that for any given element of the domain only one arrow points to an element of the range.

As a counter-example, the following diagram shows a relation which is not a function:



Certainly, what you can buy is related to how much money you have, but it is not a function of how much money you have. Looked at yet another way, knowing a function-rule is enough to make a prediction. If we know how much money a boy has, and we see the Venn diagram, can we predict what he will buy?

2.9 Experiment

PRELIMINARY PREPARATION!!! You must devise a timing device for this experiment. We suggest the following: borrow the school's tape recorder, and record the beeps from the busy signal of your telephone (you can get the busy signal by dialing your own number). Then you can play the tape loud enough for the entire class to hear. The beeps mark off one-second intervals, which should be about right.

We will follow the instructions in the teacher's guide except for this one change. No magnets will be used to hold the ball in position. Partner "A" drops the ball, and partner "B" marks its position at the next beep and on successive beeps. The first mark (and beep) is considered the start, or zero-beep-mark. We want to measure how far it has traveled from the zero-beep-mark at times of 1, 2, 3, (etc.) beeps later.

To get another trial, simply drop another ball. If you need to get the balls out of the syrup after class, see if the science department has a large magnet.

2.10 The Graph and the Equation

See the comments for Section 2.5 regarding the physical interpretation of the slope value. This one has even more meaning.

2.11 The Point-Slope Form

The text discusses a graph where the domain is limited to values greater than or equal to "a" (and "a" greater than zero); a case where there can be no "y-intercept." Physical situations where this would be the case are not overly abundant, but here's one for a start (can you think of others?):

There is a mathematical function which connects the number of paper clips to their weight in pounds. To find this function, I might use one of the produce scales in my neighborhood grocery store. The domain would be the number of paper clips placed in the pan; the range would be the pound-reading of the scale. Now the kind of scale I am thinking about has a revolving dial which lights up when a weight is placed on the pan. The trouble here is that there has to be a certain amount of weight to switch the light on. Certainly, one paper clip would not be enough to do it. It would take several paper clips (lets say 10, just to have a definite number to talk about) before the light would come on so that I could read the scale. What this means is that I could read the scale and get data for 10, 11, 12, and more paper clips, but not for less. Thus I could draw a graph, but I could not possibly have a "y-intercept" on that graph, since I couldn't possibly have points to the left of 10. Could I still find a way to write the equation of the graph line?

HINTS FOR CHAPTER THREE

3.2 Experiment

Performing the experiment described in the text is rather frustrating, since the marble keeps bouncing off of the rubber membrane. We have devised something much simpler, which will illustrate the same thing.

Tape several sheets of typing paper on a wall with masking tape. We want to mark on this paper the heights to which a ping-pong ball rebounds when it is dropped on the floor. Drop the ball one-meter from the floor (using a meter stick to get this height). Mark the one-meter height on the paper. Mark on the paper, the height to which the ball rebounds. (This last measure will, of course, be an educated guess! Repeat the trial several times -- until your marks begin to cluster into a "best height.")

Repeat the experiment, but this time mark the height to which the ball rebounds on the second bounce. Do the same for the third, fourth, and (if you can) the fifth bounce.

Measure the height-marks on your paper from the floor, and fill in a data table like the one on page 72. (If you have measured all the distances from the floor, you do <u>not</u> need a "corrected height" column -- your measured heights will be "correct."

(Note: It is impossible to get data for 10 bounces! For purposes of the discussion, give the students the data on page 70 (teacher's text) to work with. He could get data this good if he photographed the bounce heights, for example.)

No formal notes were distributed for Sections 3.3-3.10.

Appendix III

SAMPLE COMPARISONS AND GROUPING PROCEDURES

Table 1: Comparisons of Profile Vector Component Scores for Samples One and Two with the Total Population

	Sample 1 N=113	Sample 2 N=113	Total Population N=853
Math vs. Non-Math (mean change)	-0.52	-0.50	-0.31
Math Fun vs. Dull (mean change)	-0.73	-0.65	-0.63
Ideal Math Self-Concept (mean change)	-1.08	-2.06	-1.62
Actual Math Self-Concept (mean change)	-0.47	-0.42	-0.56
I Think Father Uses Math (mean change)	+0.17	+0.07	+0.08
I Would Like To Take More Math (mean change)	-0.03	-0.07	-0.07

Table 2: Error Terms for the Grouping Procedure

Sample 1 N = 113; Sample 2 N = 113

Number of Groups	Error term for Sample 1		Error term for Sample 2	
20	7.60	(increase)	7.62	(increase)
19	8.37	.77	8.55	.93
18	9.25	.88	8.93	.38
17	9.81	.56	9.76	.83
16	10.13	.32	10.92	1.16
15	11.66	1.53	11.85	.93
14	14.16	2.50	13.42	.57
13	15.67	1.51	14.22	.80
12	16.36	.69	14.75	.53
11	16.69	.33	15.58	.83
10	17.14	.45	16.09	.51
9	20.87	3.73	16.56	.47
8	21.08	.21	18.66	2.10
7	31.56	10.48	24.58	5.92
6	32.28	.72	29.99	5.41
5	44.61	12.33	41.29	11.30
4	46.00	1.39	43.10	1.81
3	51.61	5.61	56.80	13.70
2	63.08	11.47	66.62	9.82

Appendix IV

GROUPS BY TEST (8 × 1) ANALYSES OF VARIANCE

Sample One

UNIVARIATE ANOVA ON -- TEST 1: Algebra Number Properties Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	44.50	7	6.36	3.54
Within	188.44	105	1.79	
TOTAL	232.94	112		

UNIVARIATE ANOVA ON -- TEST 2: Algebra Sentences Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	34.46	7	4.92	2.57
Within	201.01	105	1.91	
TOTAL	235.47	112		

UNIVARIATE ANOVA ON -- TEST 3: Algebra Translation Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	8.90	7	1.27	1.13
Within	117.74	105	1.12	
TOTAL	126.64	112		

UNIVARIATE ANOVA ON -- TEST 4: Math vs. Non-Math Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	315.77	7	45.11	1.84
Within	2575.51	105	24.53	
TOTAL	2891.28	112		

Sample One (continued)

UNIVARIATE ANOVA ON -- TEST 5: Math Fun vs. Dull Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	338.46	7	48.35	2.08
Within	2441.41	105	23.25	
TOTAL	2779.88	112		

UNIVARIATE ANOVA ON -- TEST 6: Pro Math Composite Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	543.41	7	77.63	1.90
Within	4295.03	105	40.91	
TOTAL	4838.44	112		

UNIVARIATE ANOVA ON -- TEST 7: Math Easy vs. Hard Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	641.67	7	91.67	1.95
Within	4924.39	105	46.90	
TOTAL	5566.06	112		

UNIVARIATE ANOVA ON -- TEST 8: Ideal Math Self-Concept Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	1118.70	7	159.81	3.71
Within	4520.80	105	43.06	
TOTAL	5639.50	112		

UNIVARIATE ANOVA ON -- TEST 9: I Think F Uses M on Job Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	4.69	7	0.67	0.96
Within	73.38	105	0.70	
TOTAL	78.07	112		

Sample One (continued)

UNIVARIATE ANOVA ON -- TEST 10: I Use M Outside School Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	6.75	7	0.96	0.71
Within	141.78	105	1.35	
TOTAL	148.53	112		

UNIVARIATE ANOVA ON -- TEST 11: I Would Like to Use M Outside School Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	6.27	7	0.90	0.76
Within	123.41	105	1.18	
TOTAL	129.68	112		

UNIVARIATE ANOVA ON -- TEST 12: Facilitating Anxiety Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	220.43	7	31.49	1.16
Within	2854.01	105	27.18	
TOTAL	3074.44	112		

UNIVARIATE ANOVA ON -- TEST 13: Debilitating Anxiety Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	1019.25	7	145.61	3.36
Within	4544.19	105	43.28	
TOTAL	5563.44	112		

UNIVARIATE ANOVA ON -- TEST 14: Actual Math Self-Concept Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	868.83	7	124.12	2.52
Within	5163.17	105	49.17	
TOTAL	6032.00	112		

Sample One (continued)

UNIVARIATE ANOVA ON -- TEST 15: Orderliness Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	357.98	7	51.14	1.16
Within	4622.39	105	44.02	
TOTAL	4980.37	112		

UNIVARIATE ANOVA ON -- TEST 16: Messiness Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	361.74	7	51.68	1.63
Within	3328.75	105	31.70	
TOTAL	3690.50	112		

UNIVARIATE ANOVA ON -- TEST 17: Take More Art Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	9.96	7	1.42	2.00
Within	74.82	105	0.71	
TOTAL	84.78	112		

UNIVARIATE ANOVA ON -- TEST 18: Take More Literature Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	6.73	7	0.96	1.35
Within	74.63	105	0.71	
TOTAL	81.36	112		

UNIVARIATE ANOVA ON -- TEST 19: Take More Social Studies Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	3.04	7	0.43	0.71
Within	64.64	105	0.62	
TOTAL	67.68	112		

Sample One (continued)

UNIVARIATE ANOVA ON -- TEST 20: Take More Math Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	8.42	7	1.20	1.58
Within	79.85	105	0.76	
TOTAL	88.27	112		

UNIVARIATE ANOVA ON -- TEST 21: Take More Science Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	5.77	7	0.82	1.10
Within	78.51	105	0.75	
TOTAL	84.28	112		

UNIVARIATE ANOVA ON -- TEST 22: Necessary Arithemetic Operations

SOURCE OF VARIATION	SS	DF	MS	F
Between	12.85	7	1.84	0.39
Within	500.51	105	4.77	
TOTAL	513.36	112		

UNIVARIATE ANOVA ON -- TEST 23: Necessary Arithmetic Operations - Non-attempts

SOURCE OF VARIATION	SS	DF	MS	F
Between	26.72	7	3.82	0.65
Within	620.38	105	5.91	
TOTAL	647.10	112		

UNIVARIATE ANOVA ON -- TEST 24: Algebra Number Properties Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	22.09	7	3.16	1.71
Within	194.04	105	1.85	
TOTAL	216.12	112		

Sample One (continued)

UNIVARIATE ANOVA ON -- TEST 25: Algebra Sentences Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	10.19	7	1.46	0.80
Within	190.72	105	1.82	
TOTAL	200.90	112		

UNIVARIATE ANOVA ON -- TEST 26: Algebra Translation Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	6.47	7	0.92	0.97
Within	100.52	105	0.96	
TOTAL	106.99	112		

UNIVARIATE ANOVA ON -- TEST 27: Math vs. Non-Math Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	233.79	7	33.40	1.49
Within	2346.94	105	22.35	
TOTAL	2580.73	112		

UNIVARIATE ANOVA ON -- TEST 28: Math Fun vs. Dull Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	120.73	7	17.25	0.75
Within	2413.08	105	22.98	
TOTAL	2533.81	112		

UNIVARIATE ANOVA ON -- TEST 29: Pro Math Composite Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	266.81	7	38.12	1.01
Within	3949.76	105	37.62	
TOTAL	4216.56	112		

Sample One (continued)

UNIVARIATE ANOVA ON -- TEST 30: Math Easy vs. Hard Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	195.66	7	27.95	0.64
Within	4618.59	105	43.99	
TOTAL	4814.25	112		

UNIVARIATE ANOVA ON -- TEST 31: Ideal Math Self-Concept Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	360.61	7	51.52	1.15
Within	4711.20	105	44.87	
TOTAL	5071.81	112		

UNIVARIATE ANOVA ON -- TEST 32: I Think F Uses M on Job Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	23.95	7	3.42	3.89
Within	92.44	105	0.88	
TOTAL	116.39	112		

UNIVARIATE ANOVA ON -- TEST 33: I Use Math Outside School Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	14.23	7	2.03	1.47
Within	144.84	105	1.38	
TOTAL	159.06	112		

UNIVARIATE ANOVA ON -- TEST 34: I Would Like to Use Math Outside School Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	4.93	7	0.70	0.57
Within	129.35	105	1.23	
TOTAL	134.28	112		

Sample One (continued)

UNIVARIATE ANOVA ON -- TEST 35: Facilitating Anxiety Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	194.71	7	27.82	1.12
Within	2605.97	105	24.82	
TOTAL	2800.69	112		

UNIVARIATE ANOVA ON -- TEST 36: Debilitating Anxiety Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	504.90	7	72.13	1.71
Within	4438.29	105	42.27	
TOTAL	4943.19	112		

UNIVARIATE ANOVA ON -- TEST 37: Actual Math Self-Concept Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	632.91	7	90.42	1.86
Within	5107.34	105	48.64	
TOTAL	5740.25	112		

UNIVARIATE ANOVA ON -- TEST 38: Orderliness Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	230.05	7	32.86	1.02
Within	3384.64	105	32.23	
TOTAL	3614.69	112		

UNIVARIATE ANOVA ON -- TEST 39: Messiness Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	184.94	7	26.42	0.79
Within	3507.56	105	33.41	
TOTAL	3692.50	112		

Sample One (continued)

UNIVARIATE ANOVA ON -- TEST 40: Take More Art Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	6.30	7	0.90	1.21
Within	77.97	105	0.74	
TOTAL	84.27	112		

UNIVARIATE ANOVA ON -- TEST 41: Take More Literature Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	1.01	7	0.14	0.23
Within	66.53	105	0.63	
TOTAL	67.54	112		

UNIVARIATE ANOVA ON -- TEST 42: Take More Social Studies Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	3.32	7	0.47	0.73
Within	68.36	105	0.65	
TOTAL	71.68	112		

UNIVARIATE ANOVA ON -- TEST 43: Take More Math Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	6.18	7	0.88	1.22
Within	76.26	105	0.73	
TOTAL	82.44	112		

UNIVARIATE ANOVA ON -- TEST 44: Take More Science Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	3.12	7	0.45	0.65
Within	72.56	105	0.69	
TOTAL	75.68	112		

GROUPS BY TEST (8 × 1) ANALYSES OF VARIANCE

Sample Two

UNIVARIATE ANOVA ON -- TEST 1: Algebra Number Properties Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	11.18	7	1.60	0.79
Within	212.70	105	2.03	
TOTAL	223.88	112		

UNIVARIATE ANOVA ON -- TEST 2: Algebra Sentences Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	19.18	7	2.74	1.42
Within	203.10	105	1.93	
TOTAL	222.28	112		

UNIVARIATE ANOVA ON -- TEST 3: Algebra Translation Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	5.45	7	0.78	0.62
Within	131.10	105	1.25	
TOTAL	136.55	112		

UNIVARIATE ANOVA ON -- TEST 4: Math vs. Non-Math Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	417.46	7	59.64	2.74
Within	2284.56	105	21.76	
TOTAL	2702.02	112		

UNIVARIATE ANOVA ON -- TEST 5: Math Fun vs. Dull Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	515.61	7	73.66	3.47
Within	2226.51	105	21.20	
TOTAL	2742.12	112		

Sample Two (continued)

UNIVARIATE ANOVA ON -- TEST 6: Pro Math Composite Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	561.61	7	80.23	2.21
Within	3813.96	105	36.32	
TOTAL	4375.56	112		

UNIVARIATE ANOVA ON -- TEST 7: Math Easy vs. Hard Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	738.45	7	105.49	2.46
Within	4499.49	105	42.85	
TOTAL	5237.94	112		

UNIVARIATE ANOVA ON -- TEST 8: Ideal Math Self-Concept Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	330.60	7	47.23	0.99
Within	5002.53	105	47.64	
TOTAL	5333.12	112		

UNIVARIATE ANOVA ON -- TEST 9: I Think F Uses M on Job Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	4.97	7	0.71	0.61
Within	122.64	105	1.17	
TOTAL	127.61	112		

UNIVARIATE ANOVA ON -- TEST 10: I Use Math Outside School Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	3.26	7	0.47	0.35
Within	138.99	105	1.32	
TOTAL	142.25	112		

Sample Two (continued)

UNIVARIATE ANOVA ON -- TEST 11: I Would Like To Use Math Outside School Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	3.65	7	0.52	0.50
Within	110.28	105	1.05	
TOTAL	113.93	112		

UNIVARIATE ANOVA ON -- TEST 12: Facilitating Anxiety Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	151.14	7	21.59	0.74
Within	3075.78	105	29.29	
TOTAL	3226.92	112		

UNIVARIATE ANOVA ON -- TEST 13: Debilitating Anxiety Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	382.49	7	54.64	0.84
Within	6842.01	105	65.16	
TOTAL	7224.50	112		

UNIVARIATE ANOVA ON -- TEST 14: Actual Math Self-Concept Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	328.60	7	46.94	0.95
Within	5213.02	105	49.65	
TOTAL	5541.62	112		

UNIVARIATE ANOVA ON -- TEST 15: Orderliness Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	110.21	7	15.74	0.41
Within	4076.36	105	38.82	
TOTAL	4186.56	112		

Sample Two (continued)

UNIVARIATE ANOVA ON -- TEST 16: Messiness Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	142.41	7	20.34	0.62
Within	3435.45	105	32.72	
TOTAL	3577.86	112		

UNIVARIATE ANOVA ON -- TEST 17: Take More Art Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	5.13	7	0.73	1.04
Within	73.86	105	0.70	
TOTAL	78.99	112		

UNIVARIATE ANOVA ON -- TEST 18: Take More Literature Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	3.92	7	0.56	0.89
Within	65.92	105	0.63	
TOTAL	69.84	112		

UNIVARIATE ANOVA ON -- TEST 19: Take More Social Studies Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	6.65	7	0.95	1.35
Within	73.91	105	0.70	
TOTAL	80.57	112		

UNIVARIATE ANOVA ON -- TEST 20: Take More Math Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	12.91	7	1.84	2.85
Within	67.82	105	0.65	
TOTAL	80.73	112		

Sample Two (continued)

UNIVARIATE ANOVA ON -- TEST 21: Take More Science Post

SOURCE OF VARIATION	SS	DF	MS	F
Between	8.18	7	1.17	1.70
Within	72.38	105	0.69	
TOTAL	80.57	112		

UNIVARIATE ANOVA ON -- TEST 22: Necessary Arithmetic Operations

SOURCE OF VARIATION	SS	DF	MS	F
Between	67.16	7	9.59	1.99
Within	506.84	105	4.83	
TOTAL	574.00	112		

UNIVARIATE ANOVA ON -- TEST 23: Necessary Arithmetic Operations - Non-Attempts

SOURCE OF VARIATION	SS	DF	MS	F
Between	10.22	7	1.46	0.20
Within	764.17	105	7.28	
TOTAL	774.39	112		

UNIVARIATE ANOVA ON -- TEST 24: Algebra Number Properties Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	14.92	7	2.13	1.03
Within	217.53	105	2.07	
TOTAL	232.44	112		

UNIVARIATE ANOVA ON -- TEST 25: Algebra Sentences Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	9.95	7	1.42	0.93
Within	159.82	105	1.52	
TOTAL	169.77	112		

Sample Two (continued)

UNIVARIATE ANOVA ON -- TEST 26: Algebra Translation Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	6.31	7	0.90	0.89
Within	106.81	105	1.02	
TOTAL	113.12	112		

UNIVARIATE ANOVA ON -- TEST 27: Math vs. Non-Math Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	90.21	7	12.89	0.60
Within	2259.83	105	21.52	
TOTAL	2350.04	112		

UNIVARIATE ANOVA ON -- TEST 28: Math Fun vs. Dull Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	161.31	7	23.04	1.14
Within	2120.05	105	20.19	
TOTAL	2281.36	112		

UNIVARIATE ANOVA ON -- TEST 29: Pro Math Composite Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	217.02	7	31.00	0.81
Within	4006.55	105	38.16	
TOTAL	4223.56	112		

UNIVARIATE ANOVA ON -- TEST 30: Math Easy vs. Hard Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	477.91	7	68.27	1.55
Within	4629.15	105	44.09	
TOTAL	5107.06	112		

Sample Two (continued)

UNIVARIATE ANOVA ON -- TEST 31: Ideal Math Self-Concept Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	579.34	7	82.76	1.72
Within	5059.60	105	48.19	
TOTAL	5638.94	112		

UNIVARIATE ANOVA ON -- TEST 32: I Think F Uses M on Job Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	40.79	7	5.83	6.20
Within	98.75	105	0.94	
TOTAL	139.54	112		

UNIVARIATE ANOVA ON -- TEST 33: I Use Math Outside School Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	4.75	7	0.68	0.49
Within	145.48	105	1.39	
TOTAL	150.23	112		

UNIVARIATE ANOVA ON -- TEST 34: I Would Like To Use Math Outside School Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	4.15	7	0.59	0.53
Within	116.77	105	1.11	
TOTAL	120.92	112		

UNIVARIATE ANOVA ON -- TEST 35: Facilitating Anxiety Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	276.23	7	39.46	1.41
Within	2943.73	105	28.04	
TOTAL	3219.96	112		

Sample Two (continued)

UNIVARIATE ANOVA ON -- TEST 36: Debilitating Anxiety Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	478.27	7	68.32	1.12
Within	6415.41	105	61.10	
TOTAL	6893.69	112		

UNIVARIATE ANOVA ON -- TEST 37: Actual Math Self-Concept Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	557.33	7	79.62	1.76
Within	4748.67	105	45.23	
TOTAL	5306.00	112		

UNIVARIATE ANOVA ON -- TEST 38: Orderliness Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	155.82	7	22.26	0.57
Within	4136.43	105	39.39	
TOTAL	4292.25	112		

UNIVARIATE ANOVA ON -- TEST 39: Messiness Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	249.56	7	35.65	1.15
Within	3266.89	105	31.11	
TOTAL	3516.45	112		

UNIVARIATE ANOVA ON -- TEST 40: Take More Art Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	5.68	7	0.81	1.26
Within	67.76	105	0.65	
TOTAL	73.43	112		

Sample Two (continued)

UNIVARIATE ANOVA ON -- TEST 41: Take More Literature Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	7.01	7	1.00	1.64
Within	64.21	105	0.61	
TOTAL	71.22	112		

UNIVARIATE ANOVA ON -- TEST 42: Take More Social Studies Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	7.59	7	1.08	1.75
Within	65.19	105	0.62	
TOTAL	72.78	112		

UNIVARIATE ANOVA ON -- TEST 43: Take More Math Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	5.95	7	0.85	1.38
Within	64.51	105	0.61	
TOTAL	70.46	112		

UNIVARIATE ANOVA ON -- TEST 44: Take More Science Pre

SOURCE OF VARIATION	SS	DF	MS	F
Between	8.55	7	1.22	1.90
Within	67.38	105	0.64	
TOTAL	75.93	112		

Appendix V

ATTITUDE TRENDS

Our data documents changes but implies nothing about causality. If one assumes that the study of the Mathematics Through Science unit was primarily responsible for these attitude changes, we can further ask if some aspects of the unit were more responsible for these changes than others. In this connection, it is interesting to note that the Y-population of the NLSMA students also show a decrease in favorable attitude towards mathematics from grades seven to eleven. These changes are shown with the MTS changes in the following table. If changes in these different populations are caused by some common factor then it would be reasonable to seek the similarities between the Mathematics Through Science unit and other mathematics courses in grades seven through eleven. It seems obvious that there is much more similarity between the cognitive level of the content than between the conceptual vehicles by which content is introduced. The NLSMA Y-population and the population studied in this experiment are obviously different, and we cannot justify any formal extrapolation of data between them. Nevertheless, a gross comparison of their attitude trends does not show them to be contradictory. This very indirect evidence leads us to suspect that the attitude changes for this study were primarily caused by the rise in cognitive level of the content of the MTS unit when compared to the normally-used textbooks.

NLSMA Identif	Scale Title	NLSMA (fall) GR 7 Mean	Pre-trial Mean	Post-trial Mean	NLSMA (fall) GR 9 Mean	NLSMA GR 11 Gp 2 (non-geom) (fall)
PY009	Math vs. Non-Math	21.56	20.24	19.93	20.28	18.02
PY010	Math Fun vs. Dull	15.40	13.45	12.82	14.56	12.25
PY011	Pro-Math Composite	36.04	33.01	32.63	34.67	31.40
PY012	Math Easy vs. Hard	28.13	26.63	26.25	27.98	25.43
PY013	Ideal Math Self-Concept	33.55	31.56	29.96	31.18	29.83
PY014	I Think Father Uses Math on Job	5.27	4.46	4.52	4.34	4.24
PY015	I Use Math Outside School	3.38	3.45	3.45	3.50	3.12
PY016	I Would Like To Use Math Outside School	3.27	3.11	3.04	3.25	2.48
PY017	Facilitating Anxiety	27.19	24.70	24.67	25.27	23.56
PY018	Debilitating Anxiety	26.48	27.70	27.85	26.81	28.44
PY019	Actual Math Self-Concept	33.36	30.72	30.21	32.31	30.36
PY020	Orderliness	37.92	36.05	36.02	37.25	37.13
PY021	Messiness	20.77	21.45	21.79	21.33	21.01
PY022	Take More Art					
PY023	Take More Literature					
PY024	Take More Social Studies					
PY025	Take More Math					
PY026	Take More Science					

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